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CALCULATING TECHNICAL EFFICIENCY SCORES USING DEA IN SAMA COLLEGE

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Abstract. The efficiency of educational groups is determined by a comparison of their inputs and outputs. For multiple inputs and outputs such a comparison is not straightforward but can be accomplished using Data Envelopment Analysis. This is a technique based on linear programming which compares each unit with all the others and determines its efficiency in terms of other groups with comparable inputs and outputs. This technique is employed to evaluate the efficiency of 5 educational groups of Sama college in Ardabil.

Keywords: Data envelopment analysis, decision making unit, efficiency, production possibility set.

AMS Subject Classification: 62-07.

1. Introduction

A relatively recent technique, called Data Envelopment Analysis (DEA) and developed by Charnes, Cooper and Rhodes [1], and Banker, Charnes and Cooper [4]. They extend the concept of technical efficiency from the case of one input and one output to that of multiple inputs and outputs. This approach is based on linear programming and allows the comparison of organizational units producing the same outputs using the same inputs, declaring one or more to be efficient, and others inefficient in the sense that the inefficient organizational unit use more inputs for the outputs they produce than the efficient ones. In this study this technique is applied to inputs and outputs of 5 educational groups of Sama college in Ardabil.

This study deals with the determination of technical efficiency, which is concerned with the production of given multiple outputs given minimal multiple inputs. For single output and single input, efficiency may be expressed as input per unit of output, and various organizational units may be compared, with the one having the lowest input per unit of output declared to be efficient, and the ones with lower ratios inefficient. For multiple inputs and outputs, such a measure is no longer straightforward, as it is not obvious how the multiple inputs and outputs should be aggregated.

In this study topics are discussed: Conventional approach for measuring educational groups efficiency; Relation of the technical efficiency and DEA;

Finding efficiency by the solved a LP; Application of DEA to Sama college; Conclusions.

2. Technical efficiency and DEA

Data Envelopment Analysis privies a measure of the efficiency of a Decision Making Unit (DMU) relative to other such units, producing the same outputs with the same inputs. The units to be compared may be enterprized, banks, schools, and this study they are educational groups of Sama college. DEA is related to the concept of technical efficiency and can be considered as a generalization of efficiency measure [2].

Technical efficiency is concerned with the quantities of inputs and outputs of production processes employed by decision making units. If there is only one input and one output, a DMU is efficient if produces more per unit of input than other units. DMU's producing less than the maximum amount of output per unit of input are inefficient. To obtain a formulation that can be generalized for multiple inputs and outputs, this is reformulated as follows.

Let the DMU to be evaluated be unit o and let k be the index of all DMU's to be considered including unit o. Let y_k be the output of unit k and x_k its input. The efficiency of DMU_o is then:

$$\frac{y_o/x_o}{Max_k\,(y_k/x_k)}$$

which cannot exceed 1.

This efficiency can also be found as the solution of the linear programming problem:

$$\max_w \quad f = (y_o/x_o)w$$

s.t:

$$(y_k/x_k)w \le 1 \text{ for all k}$$

$$w \ge 0 .$$

Obviously w is determined by binding constraint(s) corresponding

$$w = \frac{1}{Max_k\,(y_k/x_k)} .$$

The following nonlinear programming problem has the same solution:

$$\max_{u,v} \quad f = \frac{y_o u}{x_o v}$$

s.t:

$$\frac{y_k u}{x_k v} \le 1 \text{ for all k.}$$

$$u, v \ge 0 .$$

u can be interpreted as the value of the output and v as the value of the input. DMU_O can select u and v to maximize its efficiency, but the constraints stipulate that u and v must be chosen such that the efficiency of each DMU, including that of DMU_O, cannot exceed 1. This problem has multiple optimal solutions, as only w = u/v matters [3].

The problem can be generalized for multiple inputs and outputs. u is the weight or value assigned to the single output. If there are multiple outputs, represented by the index i, there are corresponding weights u_i. For multiple inputs, represented by the index j, the weights are v_j. The nonlinear programming problem is then:

$$\max_{u_i, v_j} \quad f = \frac{\sum_i y_{0i} u_i}{\sum_j x_{0j} v_j}$$

s.t:

$$\frac{\sum_i y_{ki} u_i}{\sum_j x_{kj} v_j} \le 1 \text{ for all k}$$

$$u_i, v_j \ge 0 \text{ for all i, j.}$$

The interpretation of this problem is as follows. In evaluation of DMU_o, weights u_i and v_j are used for its the outputs and the inputs respectively. These weights are chosen to give the best possible efficiency rating for DMU_o, subject to the constraints that these weights result in efficiencies of all DMU's don't be exceeded 1. Note that in perfectly competitive markets the weights would be given by the equilibrium prices of inputs and outputs. For nonprofit organization such prices are not available and often no prices are available at all. Instead weights are used, which are selected to give the highest efficiency rating possible for unit o, given the performance of all DMU's considered. Such an evaluation is totaly and absolutely fair to DMU_o, because the weights replacing prices are chosen to maximize its efficiency rating.

It can be proved that the solution of this nonlinear programming problem is equivalent to that of the following linear programming problem, which will be referred to as the *primal* problem:

$$\max_{u_i, v_j} \quad f = \sum_i y_{oi} u_i$$

s.t:

$$\sum_j x_{oj} v_j = 1$$

$$\sum_i y_{ki} u_i \le \sum_j x_{kj} v_j \text{ for all k}$$
$$u_i, v_j \ge 0 \text{ for all i, j.}$$

The problem that is the dual of this linear programming problem is as follows (the *dual* problem):

$$\min_{f, l_k} \quad g = f$$
s.t:
$$\sum_k x_{ki} l_k \le x_{oi} f \text{ for all i}$$
$$\sum_k y_{kj} l_k \ge y_{oj} \text{ for all j}$$
$$l_k \ge 0 \text{ for all k.}$$

This problem can be interpreted as that of finding a linear combination of all DMU's producing at least the same output as DMU_o but using at most a fraction f of its inputs, with f to be minimized. Both the primal and dual problem can be used for computational purposes.

The formulation given above is the one given in Charnes, Cooper, and Rhodes [1], (the CCR formulation) which assumes that the productions have constant returns to scale. This is frequently not realistic. In the case of sama college , a small college may be not made comparable to a large one by simply reducing input by some factor. This is avoided in a formulation give by Banker, Charnes, Cooper [4], (the BCC formulation) allowing for increasing and decreasing returns to scale. This is achieved by adding to the dual linear programming problem the convexity constraint $\sum_k l_k = 1$ [5].

One version of a CCR model aims to minimize inputs while satisfying at least the given output levels. This is called the input-oriented model. There is another type of model called the output-oriented model that attempts to maximize outputs without requiring more of any of the observed input values.

3. Application of DEA to Sama college

This section discuses the use of CCR formulation to evaluate the efficiency of 5 educational groups in Sama college.

Of great impotence is the choice of inputs and outputs. The following items were considered:

Inputs :

1) Educational space and Number of administrative cadre
2) Number of registered students

Output :

1) Mean scores of students and Number of graduate students

inputs and outputs data given as follows:

$DMUs$	$input_1$	$input_2$
DMU_1	13	716
DMU_2	11	531
DMU_3	14	782
DMU_4	10	601
DMU_5	14	818

Table 1: Inputs data

$DMUs$	$output_1$
DMU_1	3491
DMU_2	4290
DMU_3	2190
DMU_4	11654
DMU_5	14577

Table 2: Outputs data

The results of the CCR model summarized as follows:

$DMUs$	$efficiency$
DMU_1	0.251441
DMU_2	0.416641
DMU_3	0.144423
DMU_4	1.000000
DMU_5	0.918997

Table 3: The results of the CCR model

Of the 5 DMUs, 1 has an efficiency rating of 1, so that the remaining 4 are deemed to be inefficient. The number of efficient DMUs depends on the number of inputs and outputs and how different the DMU's are in terms of these.

4. Conclusions

Data envelopment analysis was used evaluate 5 educational groups of Sama college in Ardabil with as inputs educational space, number of administrative cadre and number of registered students, and as outputs mean scores of students and number of graduate students. For CCR formulation it was found that the efficiency rating varied from 0.144423 to 1.

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Sama Kollecində DEA istifadəsinin effektivliyinin hesablanmasının texniki nəticələri

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XÜLASƏ

Təhsil qruplarının səmərəliliyi onların giriş və çıxış verilənlərinin müqayisəsi ilə müəyyən edilir. Çox girişli və çıxışlı sistemlər üçün belə müqayisə o qədər də asan deyil və DEA-dan istifadə edərək həyata keçirilə bilər. Bu üsul xətti proqramlaşdırma metoduna əsaslanır və hər bir vahidi digərləri ilə müqaisə etməklə onun başqalarına nəzərən effektivliyini təyin edir. Bu texnikadan istifadə edərək Ərdəbildə yerləşən Sama kollecdə 5 təhsil qruplarının effektivliyini qiymətləndirmək üçün istifadə olunmuşdur.

Açar sözlər: DEA, qərar vahidi, effektivlik, istehsal imkanları yığımı.

Технические результаты расчета эффективности использования DEA в колледже Сама

С. Эбади, А. Замири

РЕЗЮМЕ

Эффективность учебных групп определяется путем сравнения их входа и выхода. Для несколькими входами и выходами такое сравнение не совсем просто, но может быть выполнена с использованием DEA. Это методика основана на линейном программировании и сравнивает каждую единицу со всеми другими и определяет его эффективность с точки зрения других групп, сопоставимых входов и выходов. Этот метод используется для оценки эффективности 5 учебных групп колледжа Сама в Ардебиле.

Ключевые слова: DEA, принятие решений единицы, эффективность, набор производственных возможностей.